UNIFICATION OF SOME SEPARATION AXIOMS

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Dedicated to the memory of Professor Doğan Çoker

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Abstract

Kandil, Kerre and Nouh unified some concepts in fuzzy topological spaces by using operations. By adapting the definition of an operation and some definitions given by these authors to topological spaces, and by giving some new definitions, we have achieved some unifications related to compactness, continuity, openness and closedness of functions. Here, we will study unifications related to separation axioms, such as $T_i$ ($i = 0, 1, 2$) and $R_2$.

Key Words: Operation, Separation axiom, Continuity, Supratopology.

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1. Introduction

In [4,5], some unifications for fuzzy topological spaces were studied. Many of these definitions and results were applied to topological spaces in [7-11]. Now an attempt will be made to unify concepts related to separation.

In a topological space, $(X, \tau)$, int, cl, scl, pcl, etc., will stand for the interior, closure, semi-closure, pre-closure operations, etc. Also, for a subset $A$ of $X$, $A^o$ and $\bar{A}$ will stand for the interior of $A$ and the closure of $A$, respectively.

1.1. Definition. Let $(X, \tau)$ be a topological space. A mapping $\varphi : P(X) \to P(X)$ is called an operation on $(X, \tau)$ if $A^o \subseteq \varphi(A)$ for all $A \in P(X)$ and $\varphi(\emptyset) = \emptyset$.

The class of all operations on a topological space $(X, \tau)$ will be denoted by $O(X, \tau)$.

The operations $\varphi, \tilde{\varphi} \in O(X, \tau)$ are said to be dual if $\varphi(A) = X \setminus (\tilde{\varphi}(X \setminus A))$ (equivalently, $\tilde{\varphi}(A) = X \setminus (\varphi(X \setminus A))$) for each $A \in P(X)$.

A partial order “$\leq$” on $O(X, \tau)$ is defined by $\varphi_1 \leq \varphi_2 \iff \varphi_1(A) \subseteq \varphi_2(A)$ for each $A \in P(X)$.

An operation $\varphi \in O(X, \tau)$ is called monotonous if $\varphi(A) \subseteq \varphi(B)$ whenever $A \subseteq B$, $(A, B \in P(X))$.

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