THE BASE POINTS OF INDEFINITE QUADRATIC FORMS IN THE CYCLE AND PROPER CYCLE OF AN INDEFINITE QUADRATIC FORM

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Abstract

Let $F = (a, b, c)$ be an indefinite quadratic form of discriminant $\Delta > 0$. In the first section, we give some preliminaries from binary quadratic forms. In the second section, we derive some results concerning the base points of indefinite quadratic forms in the cycle and proper cycle of $F$ using the transformations $\tau(F) = (-a, b, -c)$, $\xi(F) = (c, b, a)$, $\chi(F) = (-c, b, -a)$, $\psi(F) = (-a, -b, -c)$, and the right neighbor $R^i(F)$ of $F$ for $i \geq 0$.

Keywords: Quadratic form, Indefinite form, Cycle, Proper cycle, Right neighbor, Base point.

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1. Introduction.

A real binary quadratic form (or just a form) $F$ is a polynomial in two variables $x$ and $y$ of the type

$$F = F(x, y) = ax^2 + bxy + cy^2$$

with real coefficients $a, b, c$. We denote $F$ briefly by $F = (a, b, c)$. The discriminant of $F$ is defined by the formula $b^2 - 4ac$ and is denoted by $\Delta = \Delta(F)$. The form $F$ is an integral form if and only if $a, b, c \in \mathbb{Z}$, and is indefinite if and only if $\Delta(F) > 0$. An indefinite quadratic form $F = (a, b, c)$ of discriminant $\Delta$ is said to be reduced if

$$\sqrt{\Delta} - 2|a| < b < \sqrt{\Delta}.$$